

Conformal Geometric Algebra

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JOIN

Join Operation	Illustration
<p>Dipole containing round points a and b.</p> $\mathbf{a} \wedge \mathbf{b} = (a_x b_y - a_y b_x) \mathbf{e}_{31} + (a_x b_z - a_z b_x) \mathbf{e}_{42} + (a_y b_z - a_z b_y) \mathbf{e}_{13}$ $+ (a_y b_z - a_z b_y) \mathbf{e}_{23} + (a_x b_z - a_z b_x) \mathbf{e}_{12} + (a_x b_y - a_y b_x) \mathbf{e}_{13}$ $+ (a_x b_z - a_z b_x) \mathbf{e}_{12} + (a_x b_y - a_y b_x) \mathbf{e}_{23}$ $+ (a_x b_z - a_z b_x) \mathbf{e}_{12} + (a_x b_y - a_y b_x) \mathbf{e}_{23}$	
<p>Line containing flat point p and round point a.</p> $\mathbf{p} \wedge \mathbf{a} = (p_x a_y - p_y a_x) \mathbf{e}_{41} + (p_x a_z - p_z a_x) \mathbf{e}_{23}$ $+ (p_x a_z - p_z a_x) \mathbf{e}_{23} + (p_x a_y - p_y a_x) \mathbf{e}_{41}$ $+ (p_x a_z - p_z a_x) \mathbf{e}_{23} + (p_x a_y - p_y a_x) \mathbf{e}_{41}$	
<p>Circle containing dipole d and round point a.</p> $\mathbf{d} \wedge \mathbf{a} = (d_x a_y - d_y a_x + d_z a_w) \mathbf{e}_{423} + (d_x a_z - d_z a_x + d_y a_w) \mathbf{e}_{431}$ $+ (d_x a_w - d_w a_x + d_y a_z) \mathbf{e}_{412} - (d_x a_y + d_y a_z + d_z a_w) \mathbf{e}_{231}$ $+ (d_x a_z - d_z a_x + d_y a_w) \mathbf{e}_{431} + (d_x a_w - d_w a_x + d_y a_z) \mathbf{e}_{412}$ $+ (d_x a_y - d_y a_x + d_z a_w) \mathbf{e}_{423} + (d_x a_z - d_z a_x + d_y a_w) \mathbf{e}_{431}$ $+ (d_x a_w - d_w a_x + d_y a_z) \mathbf{e}_{412} + (d_x a_y - d_y a_x + d_z a_w) \mathbf{e}_{423}$	
<p>Plane containing line l and round point a.</p> $\mathbf{l} \wedge \mathbf{a} = (l_x a_y - l_y a_x - l_z a_w) \mathbf{e}_{423} + (l_x a_z - l_z a_x - l_y a_w) \mathbf{e}_{431}$ $+ (l_x a_w - l_w a_x + l_y a_z) \mathbf{e}_{412} + (l_x a_y - l_y a_x - l_z a_w) \mathbf{e}_{423}$ $+ (l_x a_z - l_z a_x - l_y a_w) \mathbf{e}_{431} + (l_x a_w - l_w a_x + l_y a_z) \mathbf{e}_{412}$	
<p>Plane containing dipole d and flat point p.</p> $\mathbf{d} \wedge \mathbf{p} = (d_x p_y - d_y p_x + d_z p_w) \mathbf{e}_{423}$ $+ (d_x p_z - d_z p_x + d_y p_w) \mathbf{e}_{431}$ $+ (d_x p_z - d_z p_x + d_y p_w) \mathbf{e}_{431} + (d_x p_y - d_y p_x + d_z p_w) \mathbf{e}_{423}$ $- (d_x p_z - d_z p_x + d_y p_w) \mathbf{e}_{431} + (d_x p_y - d_y p_x + d_z p_w) \mathbf{e}_{423}$	
<p>Sphere containing circle c and round point a.</p> $\mathbf{c} \wedge \mathbf{a} = -(c_x a_y - c_y a_x + c_z a_w) \mathbf{e}_{423}$ $+ (c_x a_z - c_z a_x + c_y a_w) \mathbf{e}_{431}$ $+ (c_x a_z - c_z a_x + c_y a_w) \mathbf{e}_{431} + (c_x a_y - c_y a_x + c_z a_w) \mathbf{e}_{423}$ $+ (c_x a_z - c_z a_x + c_y a_w) \mathbf{e}_{431} + (c_x a_y - c_y a_x + c_z a_w) \mathbf{e}_{423}$ $+ (c_x a_z - c_z a_x + c_y a_w) \mathbf{e}_{431} + (c_x a_y - c_y a_x + c_z a_w) \mathbf{e}_{423}$	
<p>Sphere containing dipoles d and f.</p> $\mathbf{d} \wedge \mathbf{f} = -(d_x f_y - d_y f_x + d_z f_w) \mathbf{e}_{423} + (d_x f_z - d_z f_x + d_y f_w) \mathbf{e}_{431}$ $+ (d_x f_z - d_z f_x + d_y f_w) \mathbf{e}_{431} + (d_x f_y - d_y f_x + d_z f_w) \mathbf{e}_{423}$ $+ (d_x f_z - d_z f_x + d_y f_w) \mathbf{e}_{431} + (d_x f_y - d_y f_x + d_z f_w) \mathbf{e}_{423}$ $+ (d_x f_z - d_z f_x + d_y f_w) \mathbf{e}_{431} + (d_x f_y - d_y f_x + d_z f_w) \mathbf{e}_{423}$ $- (d_x f_z - d_z f_x + d_y f_w) \mathbf{e}_{431} + (d_x f_y - d_y f_x + d_z f_w) \mathbf{e}_{423}$	

MEET

Meet Operation	Illustration
<p>Circle where spheres s and t intersect.</p> $\mathbf{s} \vee \mathbf{t} = (s_x t_y - s_y t_x) \mathbf{e}_{423} + (s_x t_z - s_z t_x) \mathbf{e}_{431}$ $+ (s_x t_z - s_z t_x) \mathbf{e}_{431} + (s_x t_y - s_y t_x) \mathbf{e}_{423}$ $+ (s_x t_z - s_z t_x) \mathbf{e}_{431} + (s_x t_y - s_y t_x) \mathbf{e}_{423}$ $+ (s_x t_z - s_z t_x) \mathbf{e}_{431} + (s_x t_y - s_y t_x) \mathbf{e}_{423}$ $+ (s_x t_z - s_z t_x) \mathbf{e}_{431} + (s_x t_y - s_y t_x) \mathbf{e}_{423}$	
<p>Circle where sphere s and plane g intersect.</p> $\mathbf{s} \vee \mathbf{g} = (s_x g_y - s_y g_x) \mathbf{e}_{423} + (s_x g_z - s_z g_x) \mathbf{e}_{431}$ $+ (s_x g_z - s_z g_x) \mathbf{e}_{431} + (s_x g_y - s_y g_x) \mathbf{e}_{423}$ $+ (s_x g_z - s_z g_x) \mathbf{e}_{431} + (s_x g_y - s_y g_x) \mathbf{e}_{423}$ $+ (s_x g_z - s_z g_x) \mathbf{e}_{431} + (s_x g_y - s_y g_x) \mathbf{e}_{423}$	
<p>Line where planes g and h intersect.</p> $\mathbf{g} \vee \mathbf{h} = (g_x h_y - g_y h_x) \mathbf{e}_{413} + (g_x h_z - g_z h_x) \mathbf{e}_{235}$ $+ (g_x h_z - g_z h_x) \mathbf{e}_{235} + (g_x h_y - g_y h_x) \mathbf{e}_{413}$ $+ (g_x h_z - g_z h_x) \mathbf{e}_{235} + (g_x h_y - g_y h_x) \mathbf{e}_{413}$	
<p>Dipole where sphere s and circle c intersect.</p> $\mathbf{s} \vee \mathbf{c} = (s_x c_y - s_y c_x) \mathbf{e}_{423} + (s_x c_z - s_z c_x) \mathbf{e}_{431}$ $+ (s_x c_z - s_z c_x) \mathbf{e}_{431} + (s_x c_y - s_y c_x) \mathbf{e}_{423}$ $+ (s_x c_z - s_z c_x) \mathbf{e}_{431} + (s_x c_y - s_y c_x) \mathbf{e}_{423}$ $+ (s_x c_z - s_z c_x) \mathbf{e}_{431} + (s_x c_y - s_y c_x) \mathbf{e}_{423}$	
<p>Dipole where plane g and circle c intersect.</p> $\mathbf{g} \vee \mathbf{c} = (g_x c_y - g_y c_x) \mathbf{e}_{423} + (g_x c_z - g_z c_x) \mathbf{e}_{431}$ $+ (g_x c_z - g_z c_x) \mathbf{e}_{431} + (g_x c_y - g_y c_x) \mathbf{e}_{423}$ $+ (g_x c_z - g_z c_x) \mathbf{e}_{431} + (g_x c_y - g_y c_x) \mathbf{e}_{423}$	
<p>Round point centered at flat point p and contained by sphere s.</p> $\mathbf{s} \vee \mathbf{p} = (s_x p_y - s_y p_x) \mathbf{e}_{423} + (s_x p_z - s_z p_x) \mathbf{e}_{431}$ $+ (s_x p_z - s_z p_x) \mathbf{e}_{431} + (s_x p_y - s_y p_x) \mathbf{e}_{423}$ $- (s_x p_z - s_z p_x) \mathbf{e}_{431} + (s_x p_y - s_y p_x) \mathbf{e}_{423}$	

EXPANSION

Expansion Operation	Illustration
<p>Dipole containing round point a and orthogonal to sphere s.</p> $\mathbf{a} \wedge \mathbf{s} = (a_x s_y - a_y s_x) \mathbf{e}_{41} + (a_x s_z - a_z s_x) \mathbf{e}_{23}$ $+ (a_x s_z - a_z s_x) \mathbf{e}_{23} + (a_x s_y - a_y s_x) \mathbf{e}_{41}$ $+ (a_x s_z - a_z s_x) \mathbf{e}_{23} + (a_x s_y - a_y s_x) \mathbf{e}_{41}$	
<p>Dipole containing round point a and orthogonal to plane g.</p> $\mathbf{a} \wedge \mathbf{g} = (a_x g_y - a_y g_x) \mathbf{e}_{423} + (a_x g_z - a_z g_x) \mathbf{e}_{431}$ $+ (a_x g_z - a_z g_x) \mathbf{e}_{431} + (a_x g_y - a_y g_x) \mathbf{e}_{423}$ $+ (a_x g_z - a_z g_x) \mathbf{e}_{431} + (a_x g_y - a_y g_x) \mathbf{e}_{423}$	
<p>Circle containing dipole d and orthogonal to sphere s.</p> $\mathbf{d} \wedge \mathbf{s} = (d_x s_y - d_y s_x - d_z s_w) \mathbf{e}_{423} + (d_x s_z - d_z s_x - d_y s_w) \mathbf{e}_{431}$ $+ (d_x s_z - d_z s_x - d_y s_w) \mathbf{e}_{431} + (d_x s_y - d_y s_x - d_z s_w) \mathbf{e}_{423}$ $+ (d_x s_z - d_z s_x - d_y s_w) \mathbf{e}_{431} + (d_x s_y - d_y s_x - d_z s_w) \mathbf{e}_{423}$	
<p>Circle containing dipole d and orthogonal to plane g.</p> $\mathbf{d} \wedge \mathbf{g} = (d_x g_y - d_y g_x) \mathbf{e}_{423} + (d_x g_z - d_z g_x) \mathbf{e}_{431}$ $+ (d_x g_z - d_z g_x) \mathbf{e}_{431} + (d_x g_y - d_y g_x) \mathbf{e}_{423}$ $+ (d_x g_z - d_z g_x) \mathbf{e}_{431} + (d_x g_y - d_y g_x) \mathbf{e}_{423}$	
<p>Line containing flat point p and orthogonal to sphere s.</p> $\mathbf{p} \wedge \mathbf{s} = (p_x s_y - p_y s_x) \mathbf{e}_{41} + (p_x s_z - p_z s_x) \mathbf{e}_{23}$ $+ (p_x s_z - p_z s_x) \mathbf{e}_{23} + (p_x s_y - p_y s_x) \mathbf{e}_{41}$ $+ (p_x s_z - p_z s_x) \mathbf{e}_{23} + (p_x s_y - p_y s_x) \mathbf{e}_{41}$	
<p>Line containing flat point p and orthogonal to plane g.</p> $\mathbf{p} \wedge \mathbf{g} = -p_x g_y \mathbf{e}_{423} + (p_x g_z - p_z g_x) \mathbf{e}_{431}$ $+ (p_x g_z - p_z g_x) \mathbf{e}_{431} - p_x g_y \mathbf{e}_{423}$ $- p_x g_y \mathbf{e}_{423} + (p_x g_z - p_z g_x) \mathbf{e}_{431}$	

Meet Operation	Illustration
<p>Dipole where sphere s and line l intersect.</p> $\mathbf{s} \vee \mathbf{l} = (s_x l_y - s_y l_x) \mathbf{e}_{423} + (s_x l_z - s_z l_x) \mathbf{e}_{431}$ $+ (s_x l_z - s_z l_x) \mathbf{e}_{431} + (s_x l_y - s_y l_x) \mathbf{e}_{423}$ $+ (s_x l_z - s_z l_x) \mathbf{e}_{431} + (s_x l_y - s_y l_x) \mathbf{e}_{423}$	
<p>Flat point where plane g and line l intersect.</p> $\mathbf{g} \vee \mathbf{l} = (g_x l_y - g_y l_x) \mathbf{e}_{423} + (g_x l_z - g_z l_x) \mathbf{e}_{431}$ $+ (g_x l_z - g_z l_x) \mathbf{e}_{431} + (g_x l_y - g_y l_x) \mathbf{e}_{423}$ $+ (g_x l_z - g_z l_x) \mathbf{e}_{431} + (g_x l_y - g_y l_x) \mathbf{e}_{423}$	
<p>Round point contained by circles c and o.</p> $\mathbf{c} \vee \mathbf{o} = (c_x o_y - c_y o_x) \mathbf{e}_{423} + (c_x o_z - c_z o_x) \mathbf{e}_{431}$ $+ (c_x o_z - c_z o_x) \mathbf{e}_{431} + (c_x o_y - c_y o_x) \mathbf{e}_{423}$ $+ (c_x o_z - c_z o_x) \mathbf{e}_{431} + (c_x o_y - c_y o_x) \mathbf{e}_{423}$	
<p>Round point centered on line l and contained by circle c.</p> $\mathbf{c} \vee \mathbf{l} = (c_x l_y - c_y l_x) \mathbf{e}_{423} + (c_x l_z - c_z l_x) \mathbf{e}_{431}$ $+ (c_x l_z - c_z l_x) \mathbf{e}_{431} + (c_x l_y - c_y l_x) \mathbf{e}_{423}$ $+ (c_x l_z - c_z l_x) \mathbf{e}_{431} + (c_x l_y - c_y l_x) \mathbf{e}_{423}$	
<p>Round point contained by sphere s and dipole d.</p> $\mathbf{s} \vee \mathbf{d} = (s_x d_y - s_y d_x) \mathbf{e}_{423} + (s_x d_z - s_z d_x) \mathbf{e}_{431}$ $+ (s_x d_z - s_z d_x) \mathbf{e}_{431} + (s_x d_y - s_y d_x) \mathbf{e}_{423}$ $+ (s_x d_z - s_z d_x) \mathbf{e}_{431} + (s_x d_y - s_y d_x) \mathbf{e}_{423}$	
<p>Round point centered in plane g and contained by dipole d.</p> $\mathbf{g} \vee \mathbf{d} = (g_x d_y - g_y d_x) \mathbf{e}_{423} + (g_x d_z - g_z d_x) \mathbf{e}_{431}$ $+ (g_x d_z - g_z d_x) \mathbf{e}_{431} + (g_x d_y - g_y d_x) \mathbf{e}_{423}$ $+ (g_x d_z - g_z d_x) \mathbf{e}_{431} + (g_x d_y - g_y d_x) \mathbf{e}_{423}$	

Expansion Operation	Illustration
<p>Sphere containing circle c and orthogonal to sphere s.</p> $\mathbf{c} \wedge \mathbf{s} = (c_x s_y - c_y s_x - c_z s_w) \mathbf{e}_{423} + (c_x s_z - c_z s_x - c_y s_w) \mathbf{e}_{431}$ $+ (c_x s_z - c_z s_x - c_y s_w) \mathbf{e}_{431} + (c_x s_y - c_y s_x - c_z s_w) \mathbf{e}_{423}$ $+ (c_x s_z - c_z s_x - c_y s_w) \mathbf{e}_{431} + (c_x s_y - c_y s_x - c_z s_w) \mathbf{e}_{423}$	
<p>Sphere containing circle c and orthogonal to plane g.</p> $\mathbf{c} \wedge \mathbf{g} = -(c_x g_y - c_y g_x + c_z g_w) \mathbf{e}_{423}$ $+ (c_x g_z - c_z g_x + c_y g_w) \mathbf{e}_{431}$ $+ (c_x g_z - c_z g_x + c_y g_w) \mathbf{e}_{431} - (c_x g_y - c_y g_x + c_z g_w) \mathbf{e}_{423}$ $+ (c_x g_z - c_z g_x + c_y g_w) \mathbf{e}_{431} - (c_x g_y - c_y g_x + c_z g_w) \mathbf{e}_{423}$	
<p>Plane containing line l and orthogonal to sphere s.</p> $\mathbf{l} \wedge \mathbf{s} = (l_x s_y - l_y s_x + l_z s_w) \mathbf{e}_{423} + (l_x s_z - l_z s_x + l_y s_w) \mathbf{e}_{431}$ $+ (l_x s_z - l_z s_x + l_y s_w) \mathbf{e}_{431} + (l_x s_y - l_y s_x + l_z s_w) \mathbf{e}_{423}$ $+ (l_x s_z - l_z s_x + l_y s_w) \mathbf{e}_{431} + (l_x s_y - l_y s_x + l_z s_w) \mathbf{e}_{423}$	
<p>Plane containing line l and orthogonal to plane g.</p> $\mathbf{l} \wedge \mathbf{g} = (l_x g_y - l_y g_x) \mathbf{e}_{423} + (l_x g_z - l_z g_x) \mathbf{e}_{431}$ $+ (l_x g_z - l_z g_x) \mathbf{e}_{431} + (l_x g_y - l_y g_x) \mathbf{e}_{423}$ $+ (l_x g_z - l_z g_x) \mathbf{e}_{431} + (l_x g_y - l_y g_x) \mathbf{e}_{423}$	
<p>Circle containing round point a and orthogonal to circle c.</p> $\mathbf{a} \wedge \mathbf{c} = (a_x c_y - a_y c_x - a_z c_w) \mathbf{e}_{423} + (a_x c_z - a_z c_x - a_y c_w) \mathbf{e}_{431}$ $+ (a_x c_z - a_z c_x - a_y c_w) \mathbf{e}_{431} + (a_x c_y - a_y c_x - a_z c_w) \mathbf{e}_{423}$ $+ (a_x c_z - a_z c_x - a_y c_w) \mathbf{e}_{431} + (a_x c_y - a_y c_x - a_z c_w) \mathbf{e}_{423}$	
<p>Circle containing round point a and orthogonal to line l.</p> $\mathbf{a} \wedge \mathbf{l} = -a_x l_y \mathbf{e}_{423} + (a_x l_z - a_z l_x) \mathbf{e}_{431}$ $+ (a_x l_z - a_z l_x) \mathbf{e}_{431} - a_x l_y \mathbf{e}_{423}$ $- a_x l_y \mathbf{e}_{423} + (a_x l_z - a_z l_x) \mathbf{e}_{431}$	
<p>Plane containing flat point p and orthogonal to circle c.</p> $\mathbf{p} \wedge \mathbf{c} = (p_x c_y - p_y c_x) \mathbf{e}_{423} + (p_x c_z - p_z c_x) \mathbf{e}_{431}$ $+ (p_x c_z - p_z c_x) \mathbf{e}_{431} + (p_x c_y - p_y c_x) \mathbf{e}_{423}$ $+ (p_x c_z - p_z c_x) \mathbf{e}_{431} + (p_x c_y - p_y c_x) \mathbf{e}_{423}$	
<p>Plane containing flat point p and orthogonal to line l.</p> $\mathbf{p} \wedge \mathbf{l} = -p_x l_y \mathbf{e}_{423} + p_x l_z \mathbf{e}_{431} - p_x l_y \mathbf{e}_{423}$ $+ (p_x l_z - p_z l_x) \mathbf{e}_{431}$	
<p>Sphere containing dipole d and orthogonal to circle c.</p> $\mathbf{d} \wedge \mathbf{c} = (d_x c_y - d_y c_x + d_z c_w) \mathbf{e}_{423} + (d_x c_z - d_z c_x + d_y c_w) \mathbf{e}_{431}$ $+ (d_x c_z - d_z c_x + d_y c_w) \mathbf{e}_{431} + (d_x c_y - d_y c_x + d_z c_w) \mathbf{e}_{423}$ $+ (d_x c_z - d_z c_x + d_y c_w) \mathbf{e}_{431} + (d_x c_y - d_y c_x + d_z c_w) \mathbf{e}_{423}$	
<p>Sphere containing dipole d and orthogonal to line l.</p> $\mathbf{d} \wedge \mathbf{l} = (d_x l_y + d_y l_x + d_z l_w) \mathbf{e}_{423}$ $+ (d_x l_z - d_z l_x - d_y l_w) \mathbf{e}_{431}$ $+ (d_x l_z - d_z l_x - d_y l_w) \mathbf{e}_{431} + (d_x l_y + d_y l_x + d_z l_w) \mathbf{e}_{423}$ $+ (d_x l_z - d_z l_x - d_y l_w) \mathbf{e}_{431} + (d_x l_y + d_y l_x + d_z l_w) \mathbf{e}_{423}$	
<p>Sphere containing round point a and orthogonal to dipole d.</p> $\mathbf{a} \wedge \mathbf{d} = (a_x d_y - a_y d_x + a_z d_w) \mathbf{e}_{423}$ $+ (a_x d_z - a_z d_x - a_y d_w) \mathbf{e}_{431}$ $+ (a_x d_z - a_z d_x - a_y d_w) \mathbf{e}_{431} + (a_x d_y - a_y d_x + a_z d_w) \mathbf{e}_{423}$ $+ (a_x d_z - a_z d_x - a_y d_w) \mathbf{e}_{431} + (a_x d_y - a_y d_x + a_z d_w) \mathbf{e}_{423}$	
<p>Sphere containing round point a and centered at flat point p.</p> $\mathbf{a} \wedge \mathbf{p} = -a_x p_y \mathbf{e}_{423} + a_x p_z \mathbf{e}_{431} + a_x p_y \mathbf{e}_{423}$ $+ (a_x p_z - a_z p_x) \mathbf{e}_{431}$	

Flat Point p (Bivector)	Illustration
0D	
Dual	$\mathbf{p}^* = p_x \mathbf{e}_{321} - p_y \mathbf{e}_{235} - p_z \mathbf{e}_{315} - p_w \mathbf{e}_{425}$
Attitude	$\text{att}(\mathbf{p}) = \mathbf{p} \vee \mathbf{e}_{3215} = p_w \mathbf{e}_5$
Flat Bulk	$\mathbf{p} \wedge = p_x \mathbf{e}_{15} + p_y \mathbf{e}_{25} + p_z \mathbf{e}_{35} + p_w \mathbf{e}_{45}$
Flat Weight	$\ \mathbf{p}\ = p_w \mathbf{e}_{45}$
Position Norm	$\frac{\ \mathbf{p}\ }{\ \mathbf{p}\ _0} = \sqrt{\frac{p_x^2 + p_y^2 + p_z^2}{p_w^2}}$

Flat Line l (Trivector)	Illustration
1D	
Dual	$\mathbf{l}^* = l_x \mathbf{e}_{23} + l_y \mathbf{e}_{31} + l_z \mathbf{e}_{12} + l_w \mathbf{e}_{15} + l_m \mathbf{e}_{25} + l_n \mathbf{e}_{35}$
Attitude	$\text{att}(\mathbf{l}) = \mathbf{l} \vee \mathbf{e}_{2315} = l_x \mathbf{e}_{15} + l_y \mathbf{e}_{25} + l_z \mathbf{e}_{35}$
Flat Bulk	$\mathbf{l} \wedge = l_x \mathbf{e}_{235} + l_y \mathbf{e}_{315} + l_z \mathbf{e}_{125}$ (moment)
Flat Weight	$\mathbf{l} \lrcorner = l_x \mathbf{e}_{415} + l_y \mathbf{e}_{425} + l_z \mathbf{e}_{435}$ (direction)
Position Norm	$\frac{\ \mathbf{l}\ }{\ \mathbf{l}\ _0} = \sqrt{\frac{l_x^2 + l_y^2 + l_z^2}{l_w^2}}$

Flat Plane g (Quadrivector)	Illustration
2D	
Dual	$\mathbf{g}^* = -g_x \mathbf{e}_1 - g_y \mathbf{e}_2 - g_z \mathbf{e}_3 + g_w \mathbf{e}_5$
Attitude	$\text{att}(\mathbf{g}) = \mathbf{g} \vee \mathbf{e}_{3215} = g_x \mathbf{e}_{235} + g_y \mathbf{e}_{315} + g_z \mathbf{e}_{125}$
Flat Bulk	$\mathbf{g} \wedge = g_x \mathbf{e}_{2315}$ (position)
Flat Weight	$\mathbf{g} \lrcorner = g_x \mathbf{e}_{4235} + g_y \mathbf{e}_{4315} + g_z \mathbf{e}_{4125} + g_w \mathbf{e}_{3215}$ (normal)
Position Norm	$\frac{\ \mathbf{g}\ }{\ \mathbf{g}\ _0} = \sqrt{\frac{g_x^2}{g_w^2}}$

FLATS

Round Point a (Vector)	Illustration	Equation	Attitude
0D		$\mathbf{a} = p_x \mathbf{e}_1 + p_y \mathbf{e}_2 + p_z \mathbf{e}_3 + \mathbf{e}_4$	$\mathbf{p} = \text{position}$ $r = \text{radius}$
Dual		$\mathbf{a} = a_x \mathbf{e}_1 + a_y \mathbf{e}_2 + a_z \mathbf{e}_3 + a_w \mathbf{e}_4 + a_u \mathbf{e}_5$	$\text{att}(\mathbf{a}) = \mathbf{a} \vee \mathbf{e}_{3215} = a_w \mathbf{e}_5$
Carrier Point		(when $a_x = a_y = a_z = a_u = 0$)	
Center		$\text{cen}(\mathbf{a}) = \text{crr}(\mathbf{a}) \vee \mathbf{a} = a_x a_w \mathbf{e}_1 + a_y a_w \mathbf{e}_2 + a_z a_w \mathbf{e}_3 + a_u a_w \mathbf{e}_4$ (identity)	
Container		$\text{con}(\mathbf{a}) = \mathbf{a} \wedge \text{car}(\mathbf{a})^* = a_x^2 \mathbf{e}_{234} + a_y^2 \mathbf{e}_{314} + a_z^2 \mathbf{e}_{124} + a_u^2 \mathbf{e}_{1234} + a_u^2 \mathbf{e}_{234} + a_u^2 \mathbf{e}_{314} + a_u^2 \mathbf{$	